Exploring Spatial Unit Effect on Spatial Optimization

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Outline

1. Spatial Optimization's Scale Issues

2. Continuous Space Location-allocation Problem

3. The λ Interval Heuristic Approach

4. Continuous Space Demand Problem

Scale in Spatial Optimization

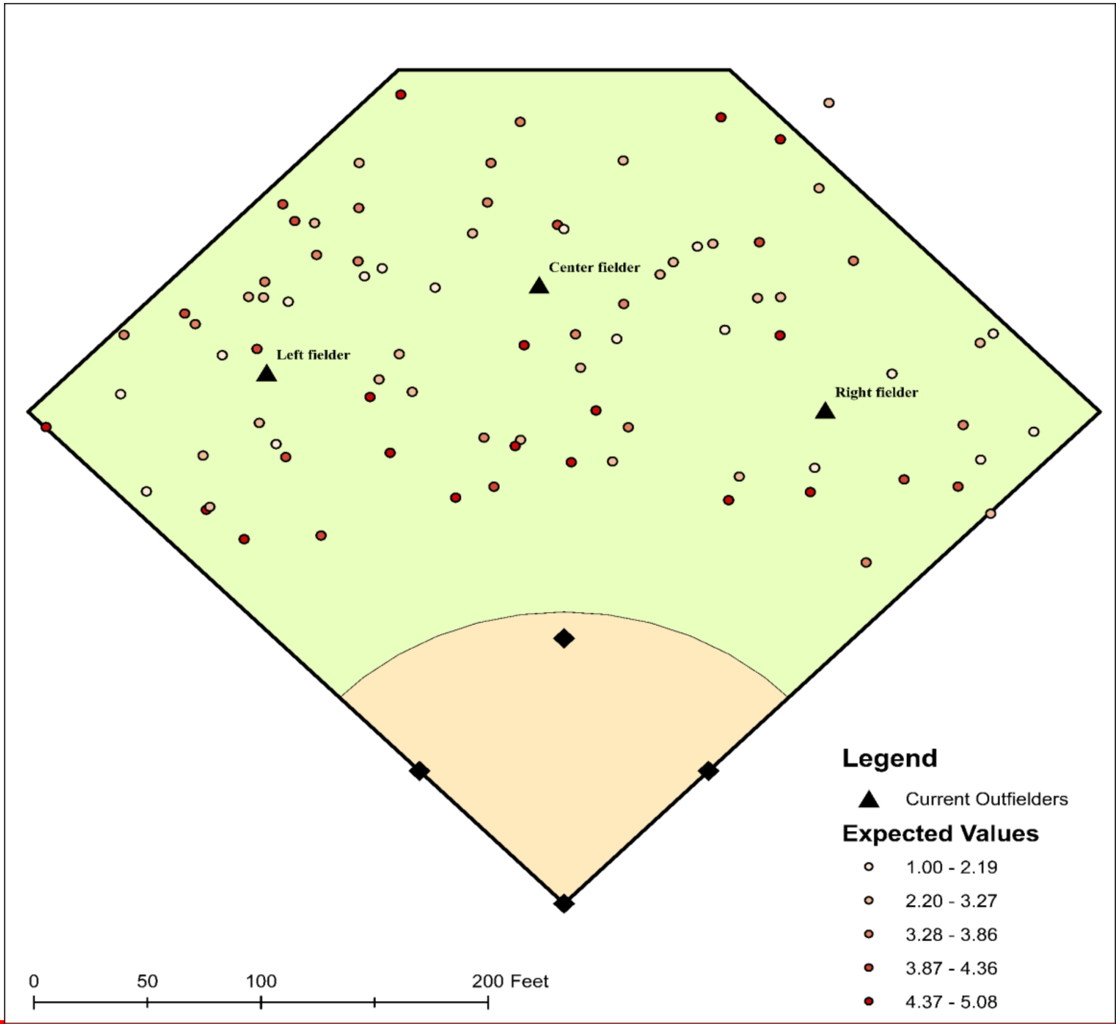
- Scale issue is also important in multi-objective spatial optimization (Openshaw and Taylor 1981; Tong and Murray, 2012)

- Solutions were highly dependent on geographical units (Fotheringham *et al.*, 1995)

- Spatial unit is related to both facility location and demand
  - Facility location
  - Demand point

Data

- UCSB Baseball team
  - Caesar Uyesaka Stadium
  - 2018-2019 season
  - 85 Batted balls (TrackMan radar system)
  - X, Y coordinates and expected values
- 3 outfielders with 90 ft coverage
- Access and coverage should be considered simultaneously



Discrete Approximation

Minimize $\sum_i \sum_j a_i d_{ij} Z_{ij}$ => Minimizing the weighted distance

Maximize $\sum_i \sum_{j \in N_i} a_i Z_{ij}$ => Maximizing covered batted balls

Subject to $\sum_j Z_{ij} = 1 \ \forall i$

$$Z_{ij} \le X_j \ \forall i,j$$

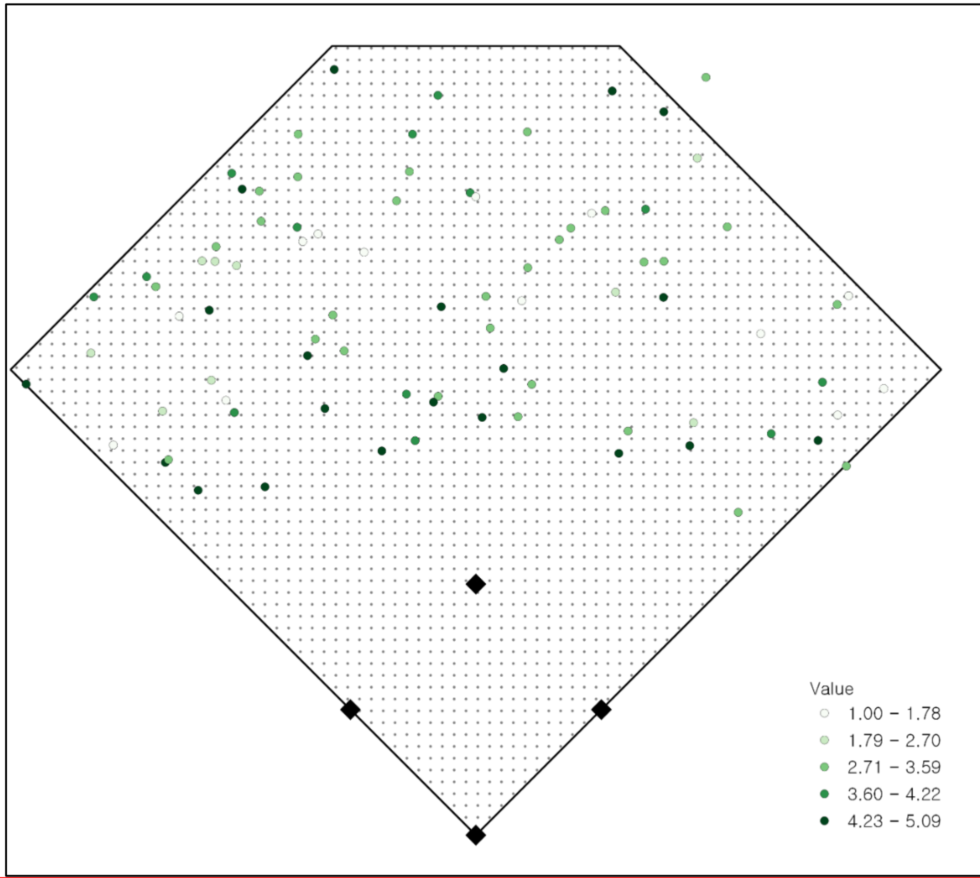
$$\sum_j X_j = p$$

$$X_j = \{0,1\} \ \forall j$$

$$Z_{ij} = \{0,1\} \ \forall i,j$$

With specific unit grids (30, 20, 10, 6, and 3 ft.)

(Pirkul and Schilling, 1991)



Problem Formularization

- Relaxation of the facility location condition

$Minimize \quad \sum_{j=1}^{p} \sum_i a_i U_{ij} \sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2}$ -> Minimizing weighted distance

$Maximize \quad \sum_i a_i Z_i$ -> Maximizing covered batted balls

$Subject\ \ to$

$$s + M(1 - Z_i) \geq U_{ij} \sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2} \quad \forall i, j$$

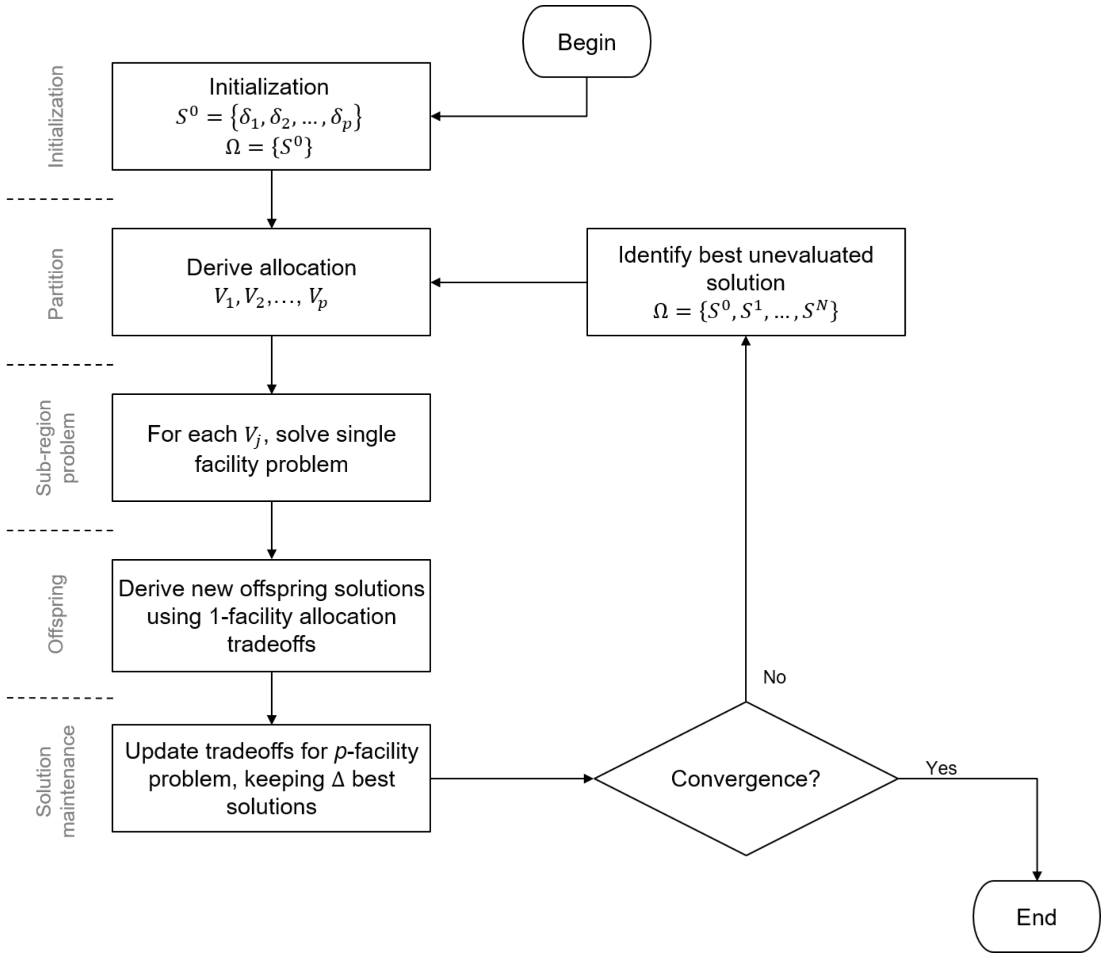
$$\sum_{j=1}^{p} U_{ij} = 1 \quad \forall i$$

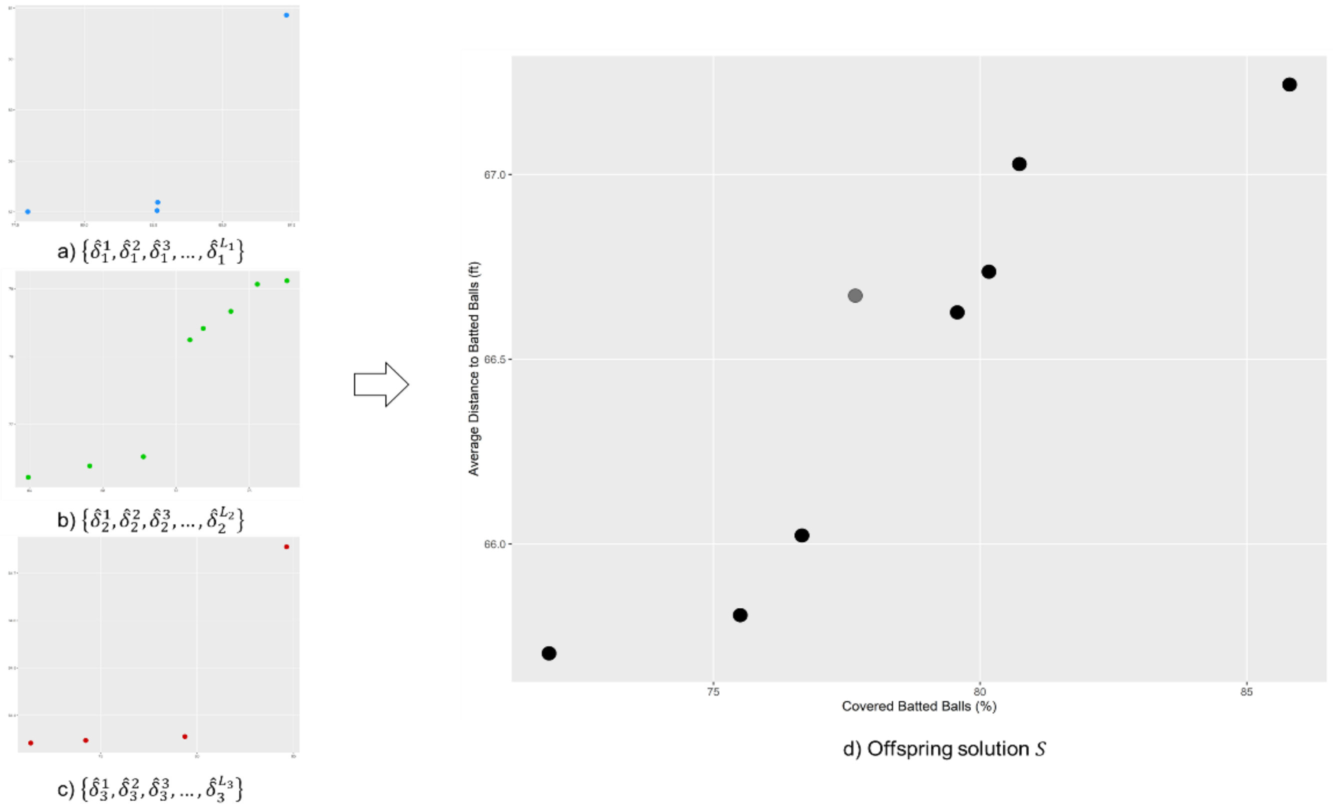
$$U_{ij} = \{0,1\} \quad \forall i, j$$

$$Z_i = \{0,1\} \quad \forall i$$

X_j, Y_j unrestricted in sign $\forall j$

Algorithm Overview





The Effect of Spatial Unit

- Potential location of facilities affect the solution set
- Trade-off between fine scale spatial unit and computation time

- The value of continuous space concept

Grid size	3	6	10	20	30	Continuous space
Max cover (%)	90.26	89.43	89.14	87.01	88.63	90.27
Min distance (ft)	63.77	63.80	63.83	64.05	64.68	61.11
The number of solutions	17	27	12	5	3	14
Computation time	49.55 hours	8.03 hours	1.95 hours	13.2 minutes	5.5 minutes	56.2 seconds

The Effect of Spatial Unit

- Compared to the current situation
- Nearing Pareto-optimal frontier
- Fine scale grid's trade-off

Empirical Bayesian Kriging

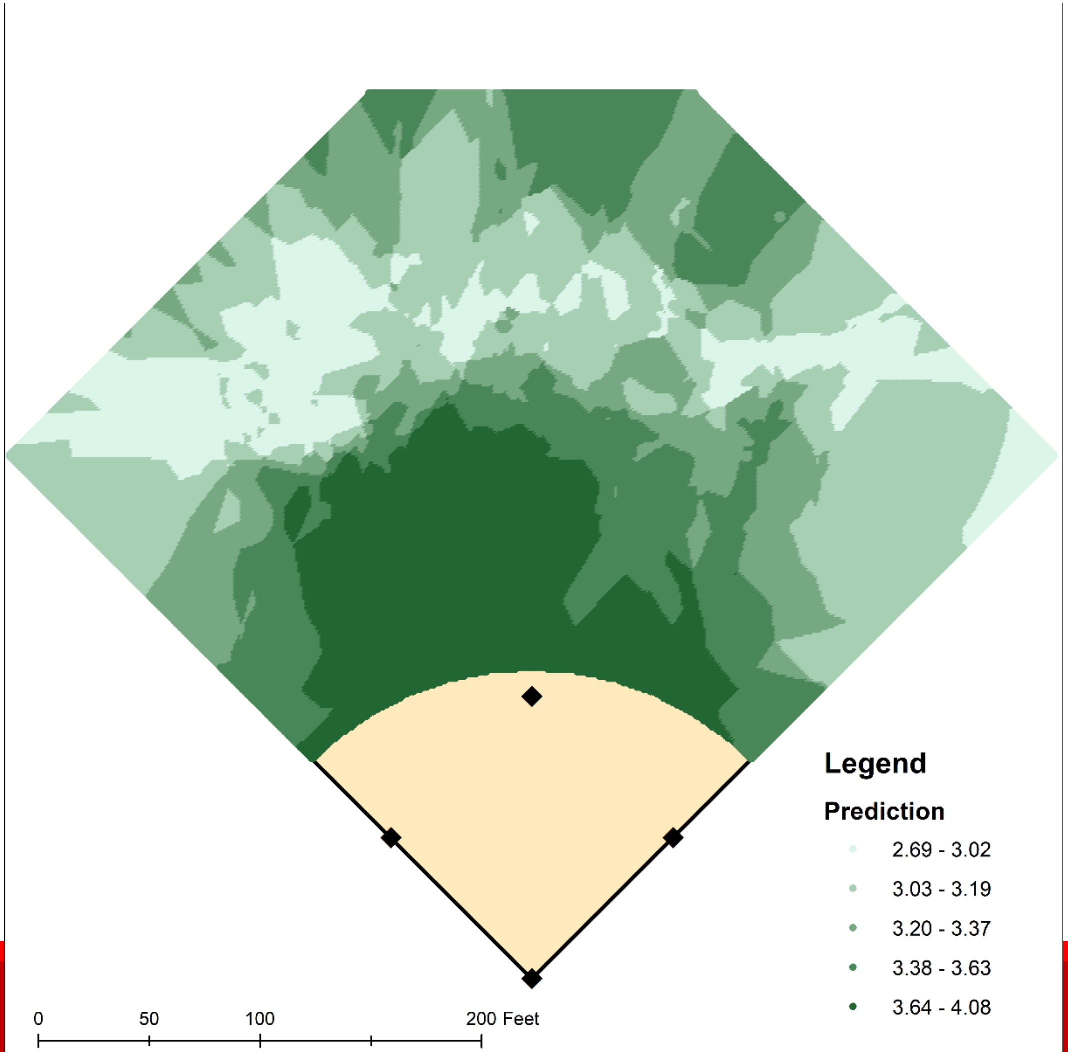
- Interpolation of certain number of sample demand points
- Implying the uncertainties on demands

- EBK: kriging method which considers the uncertainty of semi-variogram estimation

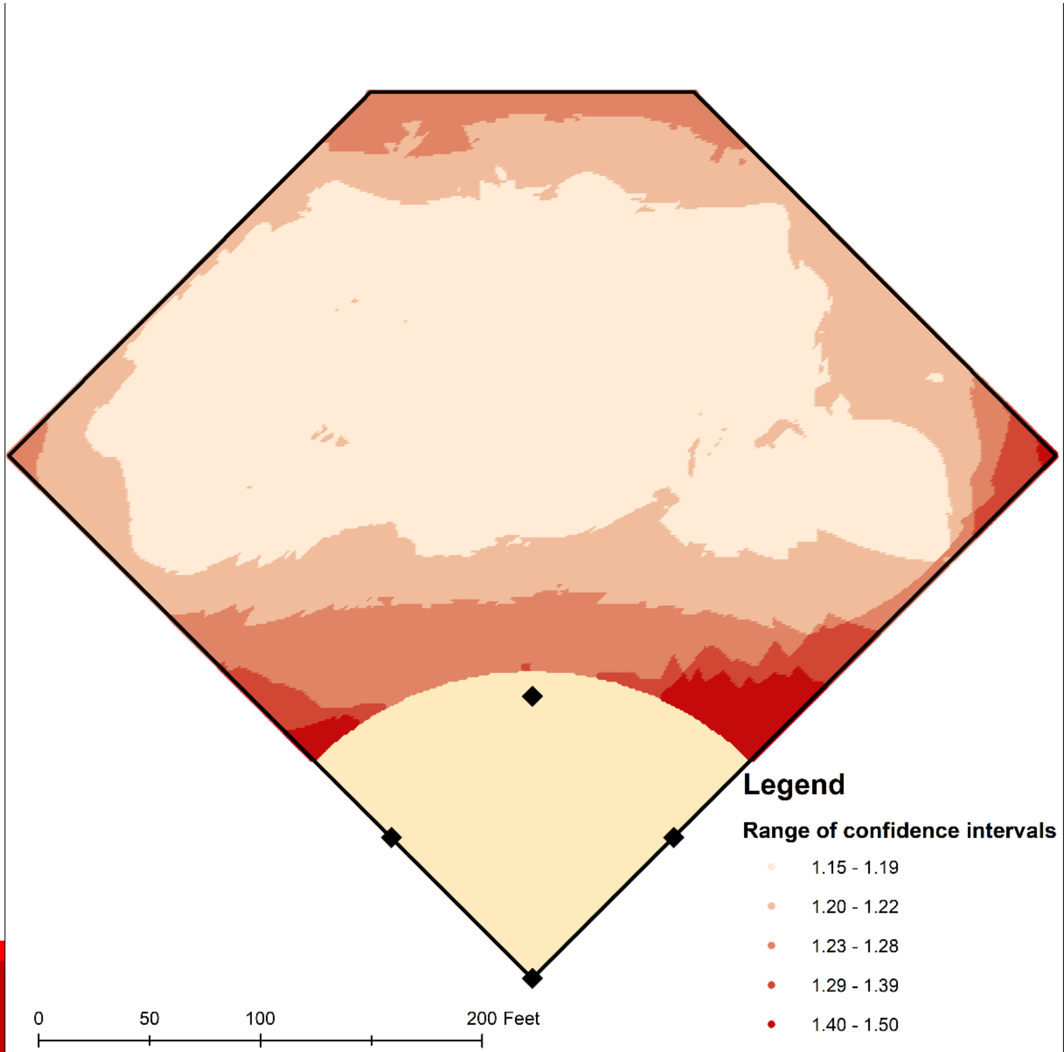
- Standard errors and confidence intervals can be calculated
- Both prediction value and confidence intervals show spatial uncertainty

Continuous Demand Representation

- EBK predicted values



- EBK 95% confidence interval's range



Summary and Future Challenges

- Spatial unit affect spatial optimization’s results
- MAUP happens on the performance of the model
- Continuous space can improve the performance of the problem
- The underlying uncertainty issues in demand interpolation

References

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Discussion and Comments