Scale Spectrum Regression

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Motivation

- Inference on the scale parameter
- Multiscale processes exist in a superposition of states
- Generalize and enhance BLISS (Stuber et al. 2017) via full scale inference

Stuber, Erica F., Lutz F. Gruber, and Joseph J. Fontaine. “A Bayesian Method for Assessing Multi-Scale Species-Habitat Relationships.” *Landscape Ecology* 32, no. 12 (December 2017): 2365–81. https://doi.org/10.1007/s10980-017-0575-y.

Setup

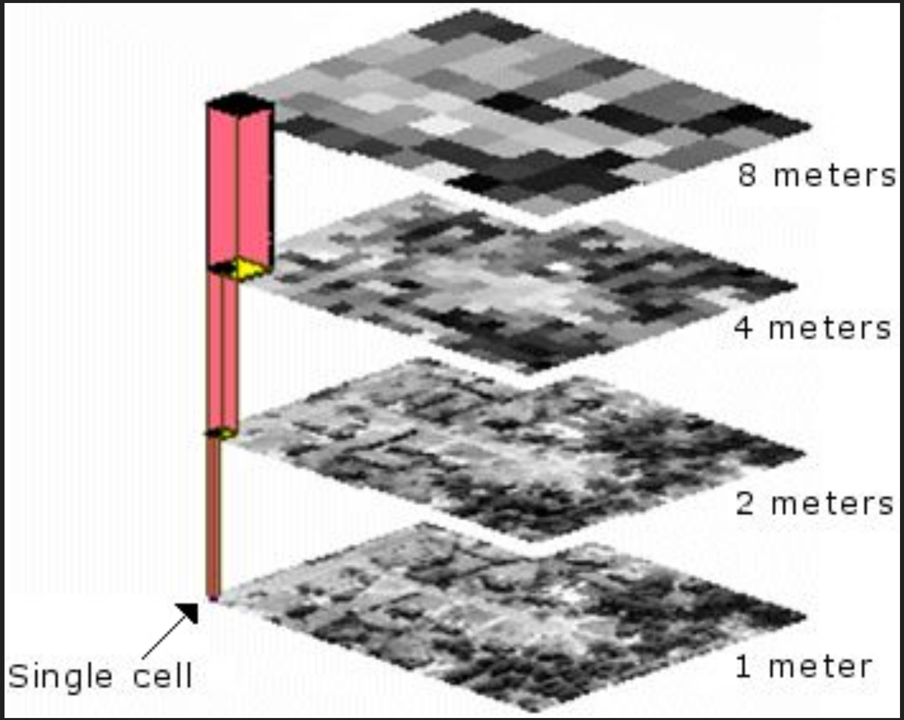
Data

Covariates

$$X \in \mathbb{R}^{N \times D}$$

Response

$$y \in \mathbb{R}^{N}$$



Intuition

- Traditional spatial analyses only allow for one scale to be considered
- Suppose there are five candidate scales. Then there are five possible combinations of those scales for analysis:
- Each scale is a larger spatial lag of X

I	II	III	IV	V
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	1	0	0	0
1	0	0	0	0

Intuition

- If we interpret multiscale processes as existing in a superposition of single-scale processes simultaneously, then we get an infinite number of combinations:

$$Z = \sum_{m=1}^{M} w_m Z[m]$$

I	II	III	IV	V
0	0	0.5	0.5	0
0	0.2	0	0.4	0.4
0.25	0.25	0.15	0	0.35
0	0.04	0.16	0	0.8
0.2	0.2	0.2	0.2	0.2
⋮	⋮	⋮	⋮	⋮

Model Components

Data

Covariates

$$X \in \mathbb{R}^{N \times D \times M}$$

Response

$$y \in \mathbb{R}^{N}$$

Parameters

Covariate effects

$$\beta \in \mathbb{R}^{D}$$

Scale parameters

$$\kappa \in \mathbb{R}^{M \times D}$$

Variance

$$\sigma^2 \in \mathbb{R}_{+}$$

Standard Normal Regression (OLS)

$$y \sim N\left(\sum_{d=1}^{D} \beta_d X_d, \sigma^2\right)$$

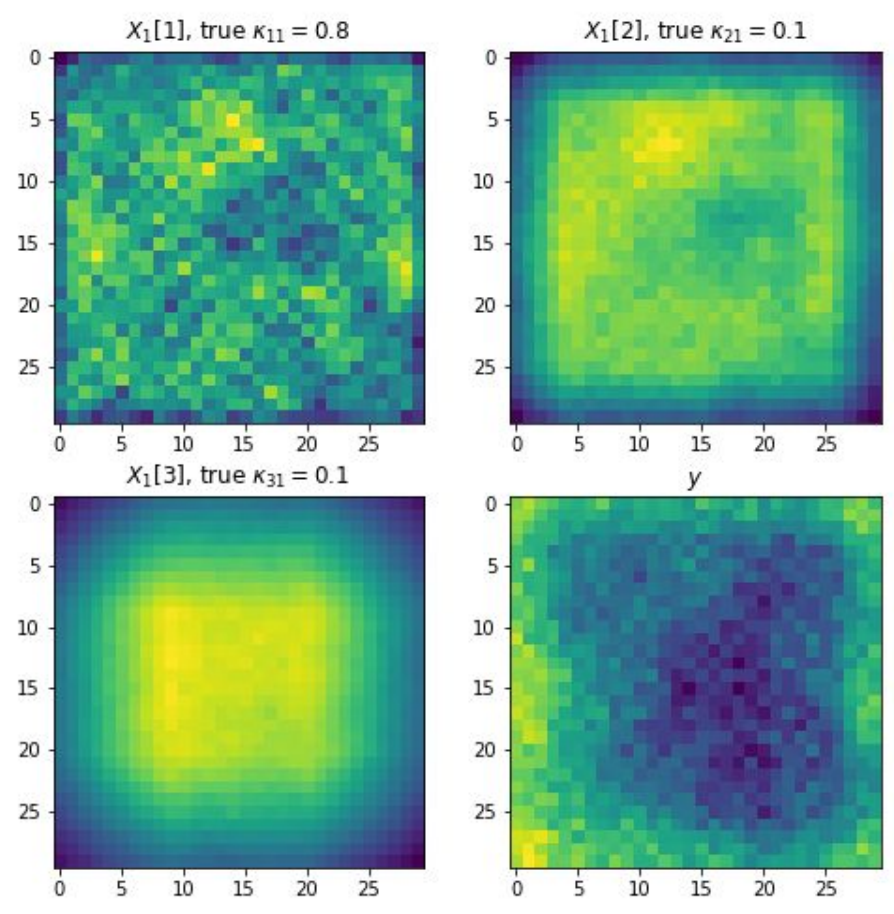
Exogenous spatial lag (SLX)

$$y \sim N\left(\sum_{d=1}^{D} \beta_d X_d + \sum_{d=1}^{D} \rho_d W X_d, \sigma^2\right)$$

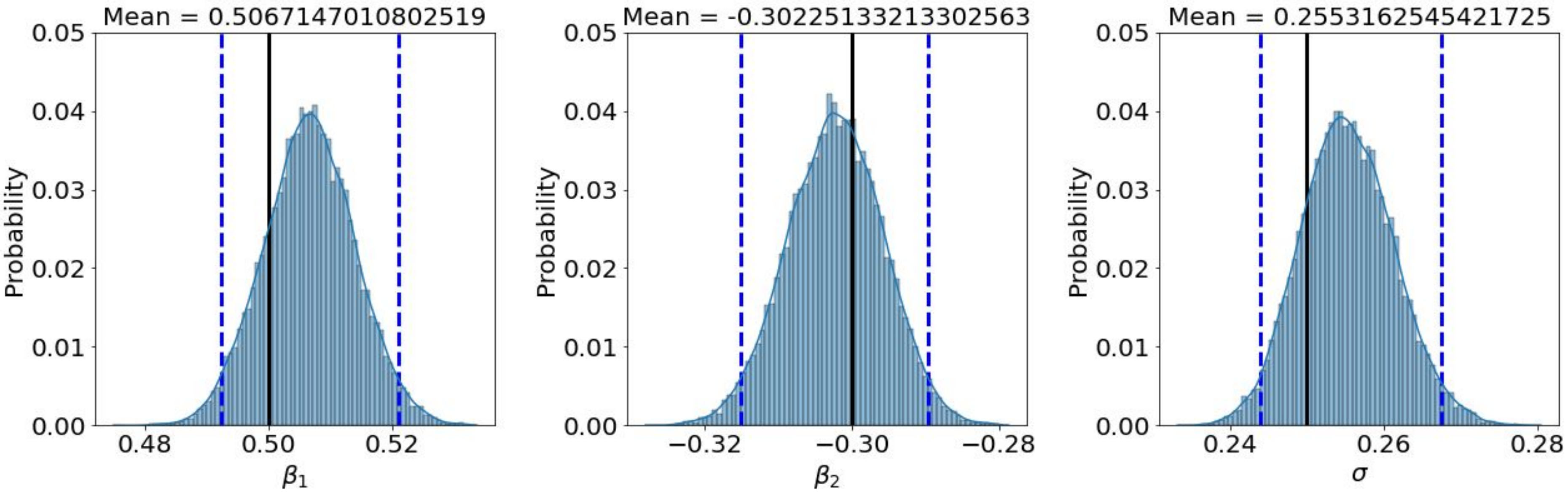
The Model

$$y \sim N\left(\sum_{d=1}^{D} \beta_d \sum_{m=1}^{M} \kappa_{md} X_d[m], \sigma^2\right)$$

Experiments: Synthetic lattice data



Experiments: Synthetic lattice data



Experiments: Synthetic lattice data

covariate	parameter	true	mean	std
1	κ_{11}	0.80	0.801253	0.006399
	κ_{21}	0.10	0.102046	0.006473
	κ_{31}	0.10	0.096701	0.003178
2	κ_{12}	0.25	0.252919	0.020719
	κ_{22}	0.50	0.501428	0.017083
	κ_{32}	0.25	0.245652	0.006993

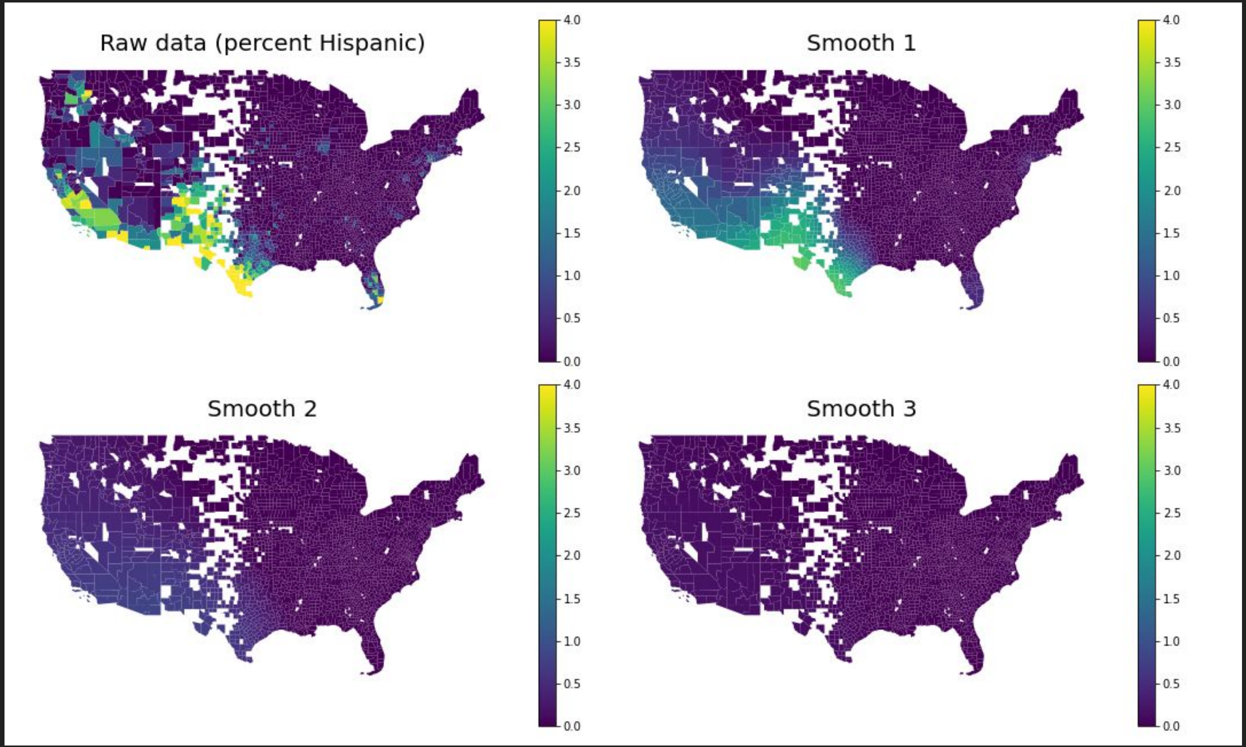
Experiments: Empirical election data

- If multiscale processes really do follow SSR's model, what information is lost when they are modeled using OLS or SLX?

Procedure:

- Fit to election data
- Generate predicted y values
- Fit OLS and SLX to those predictions

Experiments: Empirical election data

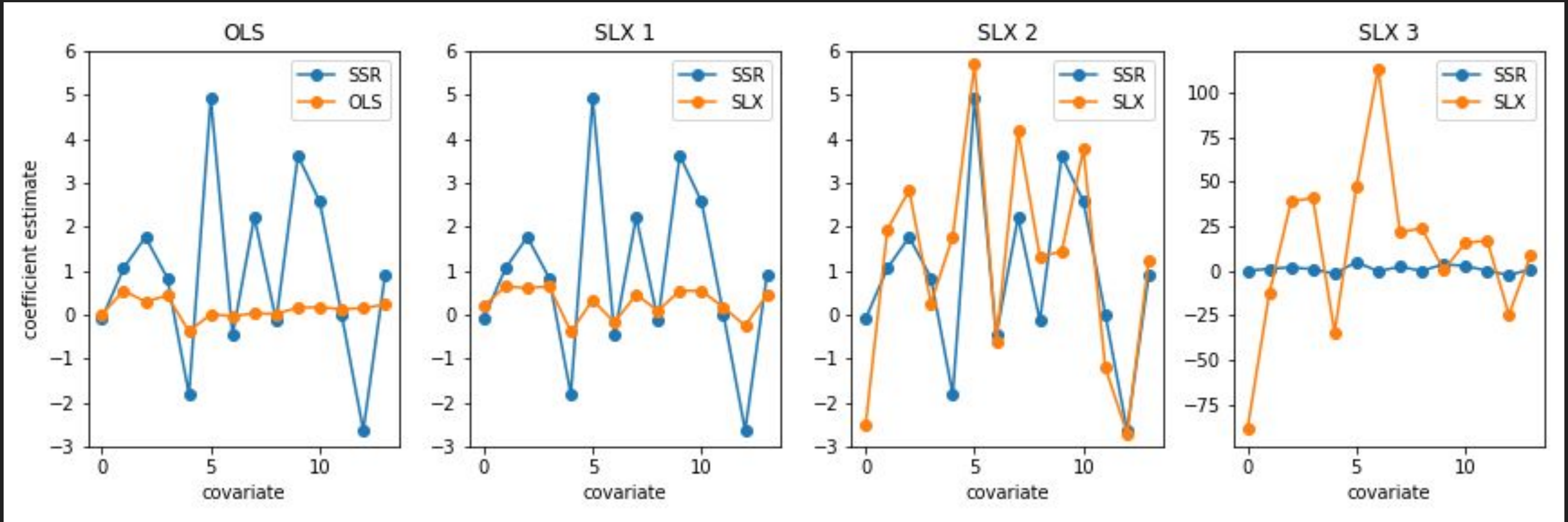


Experiments: Empirical election data

Goodness-of-fit statistic	OLS	SLX 1	SLX 2	SLX 3
R^2	0.8399	0.9856	0.9723	0.9736
AIC	2001.57	-4750.63	-2910.53	-3038.19
MSE	331.922	29.776	57.288	54.746

- Standard errors of the coefficients are all virtually zero

Experiments: Empirical election data



$$\kappa_5 = [0.01, 0.02, 0.94, 0.02]$$

Conclusion and Next Steps

- Thinking of spatial data as realizations from multiscale processes allows a different perspective on modeling scale

- Consider more applications and interpret the parameters in their contexts

- Generalize the model into SSL: Scale Spectrum *Learning* and introduce more perspectives on the model structure